On the initial condition for evolution of the perturbative QCD Pomeron in the nucleus

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Abstract. It is shown that subdominant terms found in the reggeized gluon diagram technique, to be added to Pomeron fan diagrams with the 3P interaction, can be exactly taken into account by taking the initial condition for evolution in the Glauber form. This demonstrates complete equivalence in the high-energy limit of the dipole picture and reggeized gluon technique not only on the leading level but also on the subleading level.

1 Introduction

As shown in [1, 2] the equation for propagation of the perturbative QCD pomeron in the nucleus (BK equation, [3, 4]) corresponds to the summation of fan diagrams constructed of the pomeronic Green functions connected by the splitting triple pomeron vertex V first introduced in the dipole picture by A.Mueller and B.Patel [5], afterwards derived for arbitrary number of colours from the reggeized gluon diagrams by J.Bartels and M.Wuesthoff [6] and simplified for the mulicolour case by G.P.Vacca and the author [7]. In the latter approach, however the contribution of the triple Pomeron interaction does not exhaust the total amplitude. Apart from it, there appears a contribution from the so-called 'reduced' or 'reggeized' term which is given by a single Pomeron exchange coupled to all colour dipoles in the multi-nucleon target. This contribution was explicitly introduced for 4 gluons in [6] and for 6 gluons in [8]. The reggeized contribution is obviously subdominant at high energies, so taking it into account in the BK equation is not really justified. However already at an earlier stage of the discussion of the BK equation we conjectured that one can reproduce the reggeized contribution by suitably generalizing the initial condition for the evolution. This idea was actually initiated by our discussion with Yu.Kovchegov and E.Levin of the Glauber-like initial condition proposed in [4]. In particular E.Levin rightfully noted that multiple two- gluon exchanges are not contained in the triple Pomeron interaction diagrams, and should be added via glauberization of the single-nucleon initial condition [9]. This problem is strictly speaking not important for the formulation of the BK equation nor for its actual solution, since it refers to subdominant contributions and, as calculation show, does not practically influence the solution already after evolution over some several of units of rapidity. However it is of some interest to see if the correspondence between the dipole picture and reggeized gluon technique can be extended also to this subdominant level. Also it may have some practical effect when one is interested in the evolution to not so high rapidities, when the result preserves certain trace of the chosen initial condition.

In this paper we prove that the reggeized terms which are found in the reggeized gluon technique, to be added to the triple pomeron interaction terms, are indeed exactly reproduced by choosing the Glauber-like initial condition in the BK equation.

2 Initial condition for evolution

The amplitude for the scattering of some projectile on the nucleus at rapitity y and fixed impact parameter b can be presented as

$$\mathcal{A}(y,b) = 2is \int d^2r \rho(r) \Phi(r,y,b), \tag{1}$$

where $\rho(r)$ is the colour dipole density in the projectile and $\Phi(r,y,b)$ is the amplitude for the scattering of a dipole on the nucleus. In the reggeized gluon diagram technique it is given by a sum of all fan diagrams constructed from the BFKL Green functions G and triple Pomeron vertex V for the spliiting of a Pomeron in two. For our purpose we do not need the explicit form of both, which can be found in the cited references. What we need is the lowest order term, corresponding to the single Pomeron exchange:

$$\Phi_1(r,y,b) = \int d^2r' G(y,r,r') AT(b) \rho_N(r'), \tag{2}$$

where $G(y,r,r')$ is the (forward) BFKL Green function, $AT(b)$ is the transverse nuclear density at impact parameter b and $\rho_N(r)$ is the colour dipole density of the nucleon.

To compare with the results obtained by the reggeized gluon diagram technique we introduce a Pomeron coupled to the projectile

$$P(y,r) = \int d^2r' \rho(r') G(y,r',r) \tag{3}$$

and present the reduced amplitude $\mathcal{A}/(2is)$ as

$$\tilde{\mathcal{A}}(y,b) \equiv \int d^2r \rho(r) \Phi(r,y,b) = \int d^2r P(y,r) AT(b) \rho_N(r). \tag{4}$$

Next we amputate the Pomeron $P(r)$ from the target side, that is separate the gluon propagators $1/k^4$ from its Fourier transform including them into the target impact factor, to write (4) as

$$\tilde{\mathcal{A}}(y,b) = \int d^2r D(y,r) \tau(r,b). \tag{5}$$

Here $D(y,r)$ is the amputated Pomeron coupled to the projectile and $\tau(r,b)$ is determined as a Fourier transform:

$$\tau(r,b) = AT(b) \int \frac{d^2k}{(2\pi)^2 k^4} e^{ikr} \rho_N(k). \tag{6}$$

At first sight (6) is infrared singular since the integral is divergent at $k \to 0$. However this diveregence is spurious. In fact in (6) we can subtract unity from the exponential, presenting $\tau(r)$ as

$$\tau(r,b) = \tau(0,b) + \tilde{\tau}(r,b), \tag{7}$$

where

$$\tilde{\tau}(r) = \int \frac{d^2k}{(2\pi)^2 k^4} \Big(e^{ikr} - 1 \Big) \rho_N(k) \tag{8}$$

and is infrared finite and $\tau(0)$ is infrared divergent but independent of r. Since

$$\int d^2r D(y,r) = 0, \tag{9}$$

in (5) we can drop the first term of (7) and write

$$\tilde{\mathcal{A}}(y,b) = g^2 \int d^2r \tilde{D}(r,y) \tilde{\tau}(r,b), \tag{10}$$

where we also extracted g^2 from D by writing

$$D(r,y) = g^2 \tilde{D}(r,y). \tag{11}$$

Expression (10) is obviously infrared finite.

Our next aim will be to see if appropriately changing in (10) the initial condition given by the impact factor $g^2\tilde{\tau}(r)$ we can reproduce the reggeized terms found in the analysis of the reggeized gluon diagrams. Arguments y and b will always be fixed in the following and we suppress them altogether.

3 Two scattering centers

We start from the contribution from the exchange of 4 reggeized gluons D_4 which has been studied in detail in [6, 10, 7]. It has been found that the total contribution can be separated in two terms: a term with the triple Pomeron interaction and a reggeized term $D_4^{(R)}$, which corresponds to a single Pomeron exchange coupled to both traget centers. Naturally at asymptotic energies the reggeized term is subdominant, since if Δ is the BFKL intercept (in fact the Pomeron intercept minus unity) $D_4^{(R)}$ behaves as $\exp(\Delta y)$, whereas the triple Pomeron term behaves as $\exp(2\Delta y)$.

The contribution of the reggeized term to the amplitude is given by

$$\tilde{\mathcal{A}}_4^{(R)} = \frac{1}{2}\int d^2r_1 d^2r_2 D_4^{(R)}(r_1,r_2)\tau(r_1)\tau(r_2), \tag{12}$$

where $D^{(R)}(r_1,r_2)$ is the Fourier transform of the reggeized part for two forward Pomerons:

$$D_4^{(R)}(r_1,r_2) = \int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2} e^{-ik_1r_1 - ik_2r_2} D_4^{(R)}(k_1,-k_1,k_2,-k_2). \tag{13}$$

Here $D_4^{(R)}(k_1,k_2,k_3,k_4)$ is the reggeized term as a function of gluon momenta in the high-colour limit (see [6, 7]):

$$D_4^{(R)}(1,2,3,4) = \frac{1}{2}g^2\Big(\sum_{i=1}^{4} D(i) - \sum_{i=2}^{4} D(1i)\Big), \tag{14}$$

in the shorthand notations $k_1 \equiv 1$, $k_1+k_2 \equiv 12$ etc. For our forward case, taking into account that $D(0)=0$, we find

$$D_4^{(R)}(k_1,-k_1,k_2,-k_2) = \frac{1}{2}g^2\Big(2D(k_1)+2D(k_2)-D(k_1+k_2)-D(k_1-k_2)\Big). \tag{15}$$

Performing the Fourier transform we find

$$D_4^{(R)}(r_1,r_2) = \frac{1}{2}g^2\Big(2\delta^2(r_2)D(r_1)+2\delta^2(r_1)D(r_2)-g^2\delta^2(r_1+r_2)D(r_1)-g^2\delta^2(r_1-r_2)D(r_1)\Big). \tag{16}$$

Putting this into (12) we get

$$\tilde{\mathcal{A}}_4^{(R)} = \frac{1}{2}g^2\Big\{2\tau(0)\int d^2r D(r)\tau(r) - \int d^2r D(r)\tau^2(r)\Big\}, \tag{17}$$

or, presenting $\tau(r)$ and $D(r)$ acording to (7) and (11) ,

$$\tilde{\mathcal{A}}_4^{(R)} = -\frac{1}{2}g^4\Big\{\int d^2r \tilde{D}(r)\tilde{\tau}^2(r) + \tau^2(0)\int d^2r\tilde{D}(r)\Big\}. \tag{18}$$

The last term is equal to zero, so that finally the contribution to the amplitude from the reggeized term is

$$\tilde{\mathcal{A}}_4^{(R)} = \int d^2r \tilde{D}(r)\Big[-\frac{1}{2}\Big(g^2\tilde{\tau}(r)\Big)^2\Big]. \tag{19}$$

Comparing with (10) we see that we shall reproduce this contribution if in the the lowest order term we substitute

$$g^2\tilde{\tau}(r) \to g^2\tilde{\tau}(r) - \frac{1}{2}\Big(g^2\tilde{\tau}(r)\Big)^2, \tag{20}$$

which exactly corresponds to glauberizing the initial condition:

$$g^2\tilde{\tau}(r) \to 1 - e^{-g^2\tilde{\tau}(r)}. \tag{21}$$

4 Any number of scattering centers

The preceding derivation can be easily generalized to any (even) number of gluons $n = 2p$. Using the expression for the vertex connecting a virtual photon to n gluons arranged in colourless pairs {12},{34},...{(n-1)n}, found in the linit $N_c \to \infty$ in [11], one obtains in this limit

$$D_n^{(R)}(k_1, ...k_n) = -\frac{g^{n-2}}{2^p}\int d^2r D(r) \prod_{i=1}^{n}\Big(e^{ik_i r} - 1\Big). \tag{22}$$

One can check that this expression coincides with Eq. (14) for $n = 4$ and with $D_6^{(R)}$ found in [8] for general N_c if one takes $N_c \to \infty$. In our case we have to take each target at zero momentum transfer, that is $k_1+k_2 = k_3+k_4 = ... = k_{n-1}+k_n = 0$ with p momenta $k_1, k_3, ...k_{n-1}$ as independent variables. To couple to the target we transform (22) to the coordinate space $r_1, r_3, ...r_{n-1}$:

$$D_n^{(R)}(r_1, r_3, ...r_{n-1}) = -\frac{g^{n-2}}{2^p}\int d^2r D(r) \prod_{i=1}^{p}\Big[\frac{d^2k_{2i-1}}{(2\pi)^2}\Big(2 - e^{ik_{2i-1}r} - e^{-ik_{2i-1}r}\Big)\Big]. \tag{23}$$

Contributions which come from 2 in one or several bracket factors lead to terms containing $\delta^2(r_i)$ or several such factors. After integration with the target factors $\tau(r_i)$ they will produce infrared divergent terms containing $\tau(0)$. However we know from the start that such terms should cancel, since the initial expression (22) is obviously integrable with the target factor in the infrared [1]. Knowing this we may drop 2 from the brackets in (23). Furthermore both terms remaining in the brackets will obviously give the same contribution after integration with the target factor, since the change $\mathbf{r} \to -\mathbf{r}$ does not change the latter. So effectively we get 2^p equal terms and the final result is easily found to be

$$D_n^{(R)}(r_1, r_3...r_{n-1}) = -(-1)^p g^{n-2} D(r_1) \prod_{i=2}^{p-1}\delta^2(r_{2i-1} - r_1) \tag{24}$$

(at $n = 2$ there are no δ-functions and $D_2^{(R)}(r) = D(r)$ as expected). Integrating this expression with p target factors and taking into account the symmetry factor $1/p!$ we obtain

$$\tilde{\mathcal{A}}_n^{(R)} = -(-1)^p \frac{1}{p!} g^{2p-2}\int d^2r D(r)\tau^p(r). \tag{25}$$

Eliminating the infrared divergent contributions from powers of $\tau(0)$, which, as mentioned, should be completely cancelled out, and passing to $\tilde{D}$ we finally find

$$\tilde{\mathcal{A}}_{2p}^{(R)} = -(-1)^p \frac{1}{p!} g^{2p}\int d^2r \tilde{D}(r)\tilde{\tau}^p(r). \tag{26}$$

[1] We are indebted to G.P.Vacca who drew our attention to this point

Summing the contributions for all p we obtain the Glauber expression under the sign of integral over r

$$\tilde{\mathcal{A}}^{(R)} = \sum_{p=1} \tilde{\mathcal{A}}_{2p}^{(R)} = \int d^2 r \tilde{D}(r) \Big(1 - e^{-g^2 \tilde{\tau}(r)}\Big), \tag{27}$$

which means that indeed all the reggeized contribution can be adequately taken into account by glauberizing the initial condition according to (20).

5 Conclusions

We have shown that subdominant terms found in the reggeized gluon diagram technique, to be added to the Pomeron fan diagrams with 3P interaction, can be exactly taken into account by taking the initial condition for evolution in the Glauber form. This demonstrates complete equivalence of the dipole picture and reggeized gluon technique not only on the leading level in the high-energy limit but also on the subleading level.

As mentioned, glauberization of the initial condition does not make any difference after evolution by several units of rapidity: the BK equation rapidly forgets the details of the initial condition. However the difference is certainly felt at lower stages of evolution, which may be of importance in practical applications.

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